

The *type IIA* NS5–Brane

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Abstract

The kappa-invariant worldvolume action for the NS5-brane in a D=10 type IIA supergravity background is obtained by carrying out the dimensional reduction of the M5-brane action.

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1 Introduction

The M5-brane plays an important role in studying properties of M-theory [1], the theory of strings and associated field theories. For instance, many physically important multibrane configurations, realized to be relevant to a brane description of non-Abelian gauge theories [2] and a brane-world scenario [3], can be considered as a specific compactification of a single D=11 M5-brane down to lower dimensions (with or without its subsequent T-dualization).

A direct dimensional reduction of D=11 space-time with an M5-brane down to ten-dimensional space-time produces a so-called NS5-brane of type IIA supergravity which has been intensively studied in relation to six-dimensional gauge theories [4] and “little string theories” [5]².

The verification of the quantum consistency of M-theory requires, in particular, finding a mechanism of anomaly cancellation in the presence of M5-branes. It has been shown that the anomaly problem has a natural solution in the case of $D = 10$ NS5-branes [11], while in the case of the D=11 M5-brane the situation is much more subtle and requires additional study [11, 12, 13, 14]. Mechanisms for the M5-brane anomaly cancellation proposed recently in [12] include as an important feature the reduction of the structure group $SO(5)$ of the normal bundle of the M5-brane down to its $SO(4)$ subgroup. Such a reduction implies an existence of a covariantly constant vector field and, therefore, looks very much as a dimensional reduction (to be more precise, the dimensional reduction is a particular case of such an ‘M5-brane framing’ [12])³. These facts provide us with a motivation to study in more detail the dynamical and symmetry properties of the NSIIA five-brane by constructing a full worldvolume action describing its dynamics in a type IIA D=10 supergravity background.

By now the action for the NS5-brane has been constructed up to a second order in the field strength of a two-rank self-dual worldvolume gauge field of the five-brane and only in a background of the bosonic sector of IIA D=10 supergravity [15, 16].

The aim of this paper is to get a full, nonlinear and κ -symmetric, NS5-brane action in a curved IIA D=10 target superspace by carrying out the direct dimensional reduction of the D=11 M5-brane action [6, 7, 8], and thus filling in a gap in the list of worldvolume actions for supersymmetric extended objects found in string theory.

The fact that the NS5-brane can be regarded as an M5-brane propagating in a dimensionally reduced D=11 supergravity background substantially simplifies the analysis of the NS5-brane model, in particular, allowing one to derive its symmetries and dynamical properties directly from those of the M5-brane.

²The double dimensional reduction of the M5-brane action [6, 7, 8] is well known to result in a type IIA D=10 Dirichlet 4-brane. It has also been shown [9, 10] how by reducing the M5-brane action one may arrive at a duality-symmetric D3-brane action.

³In contrast to [12] the analysis of ref. [13] is based on the assumption that a full understanding of anomaly cancellation requires keeping the full $SO(5)$. We are thankful to Jeff Harvey for clarifying this difference in the approaches.

For instance, the physical field content of the IIA D=10 NS5-brane is the same as of the M5-brane. The bosonic sector consists of three degrees of freedom corresponding to the two-rank self-dual worldvolume field and five worldvolume scalars. In the case of the M5-brane the five scalar fields describe its oscillations in a D=11 background in the directions transversal to the M5-brane worldvolume, while in the case of the NS5-brane four scalar fields correspond to transversal oscillations in a D=10 background, and the fifth scalar field (corresponding to the compactified dimension of the D=11 space) ‘decouples’ and becomes a ‘purely’ worldvolume field. This results in the abovementioned reduction of the M5-brane normal bundle structure group $SO(5)$ down to $SO(4)$. For both five-branes eight fermionic fields can be associated with brane ‘oscillations’ in Grassmann directions of corresponding target superspaces.

To get the action describing the dynamics of the physical modes of the NS5-brane as a dimensionally reduced M5-brane action we first briefly remind the structure and properties of the latter.

In Sections 2–5 we consider bosonic M5- and NS5-branes and in Section 6 we describe the full target-superspace covariant and κ -invariant NS5-brane action.

2 The M5-brane action

In the absence of interactions with antisymmetric tensor fields of $D = 11$ supergravity the action for the bosonic sector of the M5-brane has the following form [6]

$$S = - \int d^6 \xi \left[\sqrt{-\det(\hat{g}_{mn} + i\hat{H}_{mn}^*)} + \frac{\sqrt{-\hat{g}}}{4\sqrt{-\partial a \partial a}} \hat{H}^{*mn} H_{mnr} \partial^r a \right] \quad (1)$$

where

$$m, n, \dots = 0, \dots, 5;$$

are vector indices of $d = 6$ worldvolume coordinates ξ^m ,

$$\hat{m}, \hat{n}, \dots = 0, \dots, 10,$$

are vector indices of $D = 11$ target space coordinates $\hat{X}^{\hat{m}}$

$$\hat{g}_{mn} = \partial_m \hat{X}^{\hat{m}} \hat{g}_{\hat{m}\hat{n}}^{(11)} \partial_n \hat{X}^{\hat{n}}, \quad (2)$$

is the worldvolume metric induced by embedding the five-brane into a $D = 11$ gravity background with a metric $\hat{g}_{\hat{m}\hat{n}}^{(11)}(X)$ (we use the ‘almost minus’ Minkowski signature $(+ - \dots -)$),

$H_{mnl}(\xi) = 3\partial_{[m} b_{nl]}$ is the field strength of the worldvolume antisymmetric tensor field $b_{mn}(\xi)$,

$$\hat{H}_{mn}^* \equiv \frac{1}{\sqrt{-\partial a \partial a}} \hat{H}_{mnr}^* \partial^r a, \quad \hat{H}^{*mnl} = \frac{1}{3!\sqrt{-\hat{g}}} \epsilon^{mnlrsq} H_{rsq}, \quad (3)$$

$a(\xi)$ is an auxiliary scalar field ensuring the covariance of the model, and

$$\widehat{\partial a \partial a} \equiv \partial_m a \hat{g}^{mn} \partial_n a \quad (4)$$

denotes the scalar product of the $d = 6$ vector $\partial_m a$ with respect to the metric (2). In what follows the ‘hat’ over quantities indicates that they correspond to or induced by the eleven-dimensional theory.

In addition to the usual gauge symmetry of the b_2 field

$$\delta a(\xi) = 0, \quad \delta b_{mn} = 2\partial_{[m}\varphi_{n]}(\xi), \quad (5)$$

the action (1) is invariant under the following transformations [6], [7], [8]

$$\delta a(\xi) = 0, \quad \delta b_{mn} = 2\phi_{[m}(\xi)\partial_{n]}a(\xi), \quad (6)$$

$$\delta a = \varphi(\xi), \quad \delta b_{mn} = \frac{\delta a}{\sqrt{-\widehat{\partial a \partial a}}} [\hat{\mathcal{H}}^*_{mn} - H_{mnp} \hat{g}^{ps} \frac{\partial_s a}{\sqrt{-\widehat{\partial a \partial a}}}], \quad (7)$$

where

$$\hat{\mathcal{H}}^*_{mn} = -\frac{2}{\sqrt{-\hat{g}}} \frac{\delta \mathcal{L}_{DBI}}{\delta \hat{H}^{*mn}}, \quad \mathcal{L}_{DBI} \equiv \sqrt{-\det(\hat{g}_{mn} + i\hat{H}^*_{mn})}. \quad (8)$$

Note that at the linearized level, $\hat{\mathcal{H}}^*_{mn}$ defined in (8) reduces to \hat{H}^*_{mn} .

The symmetries (6) and (7) are characteristic of the covariant approach [17] to the Lagrangian description of duality-symmetric fields. They ensure the b_2 field equation of motion to reduce to a self-duality condition, as well as the connection with non-covariant formulations [18, 19]

Let us briefly describe how one derives the symmetries (6) and (7) and gets the self-duality condition [6], [17].

To this end note that the second term in the action (1) can be written in terms of differential forms

$$\int d^6\xi \mathcal{L}_1 \equiv - \int d^6\xi \sqrt{-\hat{g}} \frac{1}{4\sqrt{-\widehat{\partial a \partial a}}} \hat{H}^{*mn} H_{mnr} \partial^r a = - \int_{\mathcal{M}^6} \frac{1}{2} \hat{v} \wedge H_3 \wedge i_{\hat{v}} H_3, \quad (9)$$

where ⁴

$$\hat{v} = d\xi^m \hat{v}_m, \quad \hat{v}_k \equiv \frac{\partial_k a}{\sqrt{-\widehat{\partial a \partial a}}} \quad (10)$$

$$H_3 \equiv \frac{1}{3!} d\xi^m \wedge d\xi^n \wedge d\xi^l H_{lmn}, \quad i_{\hat{v}} H_3 \equiv \frac{1}{2} d\xi^m \wedge d\xi^n \hat{v}_k \hat{g}^{kl} H_{lmn}. \quad (11)$$

The variation of the first term in (1) with respect to the gauge field and the scalar $a(\xi)$ can be written in terms of differential forms as

$$\int d\xi^6 \delta \mathcal{L}_{DBI} \equiv \int d\xi^6 \delta \sqrt{-\det(\hat{g}_{mn} + i\hat{H}^*_{mn})} = \int_{\mathcal{M}^6} \hat{\mathcal{H}}^*_2 \wedge * \delta \hat{H}^*_2, \quad (12)$$

⁴In our notation $d\xi^{m_1} \wedge \dots \wedge d\xi^{m_6} = d^6\xi \epsilon^{m_1 \dots m_6}$.

where 2-forms $\hat{\mathcal{H}}_2^*$ and \hat{H}_2^* are constructed respectively from the tensors (8) and (3)

$$\hat{\mathcal{H}}_2^* \equiv \frac{1}{2} d\xi^m \wedge d\xi^n \hat{\mathcal{H}}_{nm}^*, \quad \hat{H}_2^* = i_{\hat{v}} * H_3 \equiv \frac{1}{2} d\xi^m \wedge d\xi^n \hat{H}_{nm}^* \quad (13)$$

and $*$ is the Hodge operation in $d = 6$ dimensions ⁵.

Using the identities

$$i_{\hat{v}} \delta \hat{v} = 0, \quad \Omega_6 \equiv -i_{\hat{v}} \Omega_6 \wedge \hat{v}, \quad *i_{\hat{v}} * H_3 = H_3 \wedge \hat{v}, \quad (14)$$

$$\hat{v} \wedge H_3 \wedge i_{\hat{v}} \delta H_3 = \hat{v} \wedge i_{\hat{v}} H_3 \wedge \delta H_3 + H_3 \wedge \delta H_3, \quad (15)$$

$$\hat{v} \wedge H_3 \wedge i_{\delta \hat{v}} H_3 = \delta \hat{v} \wedge * H_3 \wedge i_{\hat{v}} * H_3 = -\delta \hat{v} \wedge \hat{v} \wedge i_{\hat{v}} * H_3 \wedge i_{\hat{v}} * H_3 \quad (16)$$

and

$$\hat{v} \wedge \hat{\mathcal{H}}_2^* \wedge \hat{\mathcal{H}}_2^* = \hat{v} \wedge \hat{H}_2^* \wedge \hat{H}_2^* \iff \epsilon^{abcdef} \hat{\mathcal{H}}_{bc}^* \hat{\mathcal{H}}_{de}^* \hat{v}_f = \epsilon^{abcdef} \hat{H}_{bc}^* \hat{H}_{de}^* \hat{v}_f, \quad (17)$$

one can rewrite the variation of the Lagrangian (1) in the form

$$\int d\xi^6 \delta \mathcal{L} \equiv - \int_{\mathcal{M}^6} \left(\frac{1}{2} H_3 \wedge \delta H_3 - da \wedge \mathcal{F}_2 \wedge \delta H_3 - \frac{1}{2} d\delta a \wedge da \wedge \mathcal{F}_2 \wedge \mathcal{F}_2 \right), \quad (18)$$

where

$$\mathcal{F}_2 \equiv \frac{1}{\sqrt{-\widehat{\partial a \partial a}}} (\hat{\mathcal{H}}^* - i_{\hat{v}} H_3) = \frac{1}{2} d\xi^m \wedge d\xi^n \mathcal{F}_{nm}, \quad (19)$$

or

$$\mathcal{F}_{mn} \equiv \frac{1}{\sqrt{-\widehat{\partial a \partial a}}} (\hat{\mathcal{H}}_{mn}^* - H_{mnl} \hat{g}^{lk} \frac{\partial_k a}{\sqrt{-\widehat{\partial a \partial a}}}). \quad (20)$$

Since $H_3 = db_2$, the variation (18) can be written (up to a total derivative) in the following form ⁶

$$\int d\xi^6 \delta \mathcal{L} \equiv - \int_{\mathcal{M}^6} d(da \wedge \mathcal{F}_2) \wedge (\delta b_2 - \delta a \mathcal{F}_2), \quad (21)$$

⁵To have $** = I$ we define

$$\begin{aligned} * \Omega_2 &= -\frac{1}{2!4!} d\xi^{m_4} \wedge \dots \wedge d\xi^{m_1} \sqrt{-g} \epsilon_{m_1 \dots m_4 n_1 n_2} \Omega^{n_1 n_2}, \\ * \Omega_4 &= +\frac{1}{2!4!} d\xi^{m_2} \wedge d\xi^{m_1} \sqrt{-g} \epsilon_{m_1 m_2 n_1 \dots n_4} \Omega^{n_1 \dots n_4} \equiv \frac{1}{2!4!} d\xi_{m_2} \wedge d\xi_{m_1} \frac{1}{\sqrt{-g}} \epsilon^{m_1 m_2 n_1 \dots n_4} \Omega_{n_1 \dots n_4} \end{aligned}$$

⁶We use conventions where external derivative acts from the right:

$$d\Omega_q = \frac{1}{q!} d\xi^{m_q} \wedge \dots \wedge d\xi^{m_1} \wedge d\xi^n \partial_n \Omega_{m_1 \dots m_q}, \quad d(\Omega_p \wedge \Omega_q) = \Omega_p \wedge d\Omega_q + (-)^q d\Omega_p \wedge \Omega_q.$$

from which the invariance of the action under (6) and (7) becomes evident.

From (21) it also follows that the equation of motion of b_2 field is

$$d(da \wedge \mathcal{F}_2) = 0, \quad (22)$$

and the equation of motion of $a(x)$ is a consequence of eq. (22). It can be shown [17] that, using the symmetry (6), the second-order equation (22) reduces to the first-order self-duality condition

$$\mathcal{F}_2 \equiv \frac{1}{\sqrt{-\widehat{\partial a \partial a}}} (\hat{\mathcal{H}}^* - i_{\hat{v}} H_3) = 0, \quad (23)$$

or in components

$$\hat{\mathcal{H}}_{mn}^* = H_{mnl} \hat{g}^{lk} \frac{\partial_k a}{\sqrt{-\widehat{\partial a \partial a}}}. \quad (24)$$

To prove this note that \hat{H}_2^* is invariant under the transformations (6)

$$\begin{aligned} \delta b_2 &= da \wedge \phi_1 \equiv \sqrt{-\widehat{\partial a \partial a}} \hat{v} \wedge \phi_1 \quad \Rightarrow \\ \delta \hat{H}_2^* &\equiv \delta i_{\hat{v}} (*\hat{H}_3) = \sqrt{-\widehat{\partial a \partial a}} i_{\hat{v}} (*(\hat{v} \wedge d\phi_1)) \equiv 0. \end{aligned}$$

Hence, the transformations of the two-form (19) reduce to

$$\delta \mathcal{F}_2 = -\frac{1}{\sqrt{-\widehat{\partial a \partial a}}} \delta i_{\hat{v}} H_3 = -i_{\hat{v}} (\hat{v} \wedge d\phi_1). \quad (25)$$

Eq. (25) is simplified when one takes into account that $i_{\hat{v}} da = \sqrt{-\widehat{\partial a \partial a}} i_{\hat{v}} \hat{v} = -\sqrt{-\widehat{\partial a \partial a}}$. Then

$$\delta \mathcal{F}_2 = -d\phi_1 + i_{\hat{v}} d\phi_1 \wedge \hat{v},$$

and

$$\delta(da \wedge \mathcal{F}_2) = -da \wedge d\phi_1. \quad (26)$$

We now observe that eq. (26) is similar to the general solution of eq. (22) for $da \wedge \mathcal{F}_2$. This means that the general solution of eq. (22) can be gauged to zero with the use of the symmetry (6), and eq. (23) appears just as a result of such gauge fixing.

Remember that $\hat{\mathcal{H}}_{mn}^*$ is defined in (8) and reduces to $\hat{H}_{mn}^* = H_{mnl}^* \hat{g}^{lk} \frac{\partial_k a}{\sqrt{-\widehat{\partial a \partial a}}}$ at the linearized level, the equation (24) becoming the conventional self-duality condition $\hat{H}_{lmn}^* = H_{lmn}$. Further details on the classical dynamics of the M5-brane the reader may find in [6, 7, 8], [20]–[24].

3 Dimensional reduction of $D = 11$ gravity and the NS5-brane action

The procedure of the direct dimensional reduction assumes a compactification of some of target-space spatial dimensions (one in our case), the worldvolume of the p -brane being not compactified. A standard (string frame) ansatz for the target-space vielbein under the Kaluza-Klein reduction of one spatial dimension has the following form

$$\begin{aligned} E^{\hat{a}} &= (E^a, E^{10}) \equiv d\hat{X}^{\hat{m}} E_{\hat{m}}^{\hat{a}}(\hat{X}), & \hat{X}^{\hat{m}} &= (X^{\underline{m}}, y), & y &= \hat{X}^{10}, \\ E^a &= e^{-\frac{1}{3}\Phi} dX^{\underline{m}} e_{\underline{m}}^a(X), & E^{10} &= e^{\frac{2}{3}\Phi} (dy - dX^{\underline{m}} A_{\underline{m}}) \equiv e^{\frac{2}{3}\Phi} \mathcal{F}, \\ e_{\hat{m}}^{\hat{a}} &= \begin{pmatrix} e^{-\frac{1}{3}\Phi} e_{\underline{m}}^a & -e^{\frac{2}{3}\Phi} A_{\underline{m}} \\ 0 & e^{\frac{2}{3}\Phi} \end{pmatrix}, \end{aligned} \quad (27)$$

where y is the coordinate compactified into a torus, and the reduction means that the background fields, such as components of (27), do not depend on y which is now considered as an intrinsic scalar field in the 5-brane worldvolume. $\Phi(X)$ is the dilaton field and $A_{\underline{m}}(X)$ is the Abelian vector gauge field of $D = 10$ IIA supergravity. The $U(1)$ -gauge transformations of $A_{\underline{m}}(X)$ and y are

$$\delta A_{\underline{m}}(X) = \partial_{\underline{m}} \varphi^{(0)}(X), \quad \delta y = \varphi^{(0)}(X). \quad (28)$$

This ansatz leads to the following expression for the $D = 11$ target space metric in terms of the $D = 10$ metric $g_{\underline{mn}}^{(10)}(X) = e_{\underline{m}}^a e_{\underline{n}a}$, $A_{\underline{m}}(X)$ and $\Phi(X)$

$$\hat{g}_{\hat{m}\hat{n}}^{(11)} = \begin{pmatrix} e^{-\frac{2}{3}\Phi} (g_{\underline{mn}}^{(10)} - e^{2\Phi} A_{\underline{m}} A_{\underline{n}}) & e^{\frac{4}{3}\Phi} A_{\underline{m}} \\ e^{\frac{4}{3}\Phi} A_{\underline{n}} & -e^{\frac{4}{3}\Phi} \end{pmatrix} \quad (29)$$

and, consequently, to the following form of the six-dimensional induced metric (2)

$$\hat{g}_{mn} = e^{-\frac{2}{3}\Phi} (g_{mn} - e^{2\Phi} \mathcal{F}_m \mathcal{F}_n). \quad (30)$$

In (30)

$$g_{mn} = \partial_m X^{\underline{m}} g_{\underline{mn}}^{(10)}(X) \partial_n X^{\underline{n}}, \quad \underline{m} = 0, \dots, 9 \quad (31)$$

is the six-dimensional metric induced by embedding the 5-brane worldvolume into the ten-dimensional curved space-time and

$$\mathcal{F}_m = \partial_m y - A_m, \quad (32)$$

where $A_m = \partial_m X^{\underline{m}} A_{\underline{m}}(X)$ is the worldvolume pullback of $A_{\underline{m}}(X)$ and \mathcal{F}_m is the pullback of the one-form \mathcal{F} introduced in (27).

\mathcal{F}_m defined in (32) can be considered as a field strength of the worldvolume scalar field $y(\xi)$. It is invariant under the $U(1)$ gauge transformations (28).

In what follows we will also use an expression for the inverse worldvolume metric

$$\hat{g}^{mn} = e^{\frac{2}{3}\Phi} (g^{mn} + \frac{e^{2\Phi} \mathcal{F}^m \mathcal{F}^n}{1 - e^{2\Phi} \mathcal{F}^2}). \quad (33)$$

The NS5-brane action follows from the M5-action (1) with the background metric having a particular form (29) and the coordinate $\hat{X}^{10} = y(\xi)$ being considered as an intrinsic worldvolume scalar field. To present the explicit form of the NS5-brane action we should rewrite all its constituents in terms of $D = 10$ fields, and to ‘rescale’ worldvolume fields and their scalar products with respect to the worldvolume induced metric (31).

For instance, the Hodge duality (3) is now redefined with respect to the metric (31)

$$\hat{H}^{*mnp} = \sqrt{\frac{g}{\hat{g}}} H^{*mnp}, \quad H^{*mnl} = \frac{1}{3! \sqrt{-g}} \epsilon^{mnlrsd} H_{rsd}, \quad (34)$$

and the M5-brane field strength \hat{H}^{*mn} (3) is related to its NS5-brane counterpart H^{*mn} as

$$\hat{H}^{*mn} = \sqrt{\frac{g}{\hat{g}}} \sqrt{\frac{(\partial a)^2}{\partial a \partial a}} H^{*mn}, \quad H^{*mn} = \frac{1}{3! \sqrt{-g}} \epsilon^{kmnpqr} H_{pqr} \frac{\partial_k a}{\sqrt{-(\partial a)^2}}, \quad (35)$$

where the scalar product (4) has also been correspondingly redefined

as

$$\widehat{\partial a \partial a} \equiv \partial_k a \hat{g}^{ks} \partial_s a = e^{\frac{2}{3}\Phi} \mathcal{N}^2 (\partial a)^2, \quad (\partial a)^2 \equiv \partial_l a g^{lm} \partial_m a, \quad (36)$$

with \mathcal{N} standing for

$$\mathcal{N} \equiv \sqrt{1 + \frac{e^{2\Phi} (\mathcal{F} \partial a)^2}{(\partial a)^2 (1 - e^{2\Phi} \mathcal{F}^2)}} = e^{-\frac{1}{3}\Phi} \sqrt{\frac{\widehat{\partial a \partial a}}{(\partial a)^2}}. \quad (37)$$

In view of eqs. (30), (35), (36) and (37) the antisymmetric tensor entering the DBI-like part of the M5-brane action is reexpressed in terms of H^{*lm} as follows

$$\hat{H}_{mn}^* = \hat{g}_{ml} \hat{g}_{nk} \hat{H}^{*lk} = \hat{g}_{ml} \hat{g}_{nk} \sqrt{\frac{g}{\hat{g}}} e^{-\frac{1}{3}\Phi} \mathcal{N}^{-1} H^{*lk}, \quad \hat{g}_{ml} = e^{-\frac{2}{3}\Phi} (g_{ml} - e^{2\Phi} \mathcal{F}_m \mathcal{F}_l). \quad (38)$$

As a result, substituting (30)–(38) into the action (1), we get the action for a bosonic 5-brane coupled to the metric, the dilaton and the gauge vector field of type IIA $D = 10$ supergravity

$$S = - \int d^6 \xi e^{-2\Phi} \sqrt{-\det(g_{mn} - e^{2\Phi} \mathcal{F}_m \mathcal{F}_n)} \sqrt{\det \left(\delta_m^{n} + i \frac{e^\Phi (g_{mp} - e^{2\Phi} \mathcal{F}_m \mathcal{F}_p)}{\mathcal{N} \sqrt{\det(\delta_m^{n} - e^{2\Phi} \mathcal{F}_m \mathcal{F}_n)}} H^{*np} \right)} \\ - \frac{1}{4} \int d^6 \xi \sqrt{-g} \frac{1}{\mathcal{N}^2} H^{*mn} H_{mnk} \left(g^{kp} + \frac{e^{2\Phi} \mathcal{F}^k \mathcal{F}^p}{1 - e^{2\Phi} \mathcal{F}^2} \right) \frac{\partial_p a}{\sqrt{-(\partial a)^2}}. \quad (39)$$

Since the action (39) is nothing but the M5-brane action for a special choice of the $D = 11$ metric (29), its variation with respect to the gauge field $b_2(\xi)$ and the auxiliary scalar $a(\xi)$ has the form of eq. (21), and hence (39) is also invariant under the symmetries (6) and (7) which, as we have seen, produce the self-duality condition (24).

To rewrite the transformations and the self-duality condition (24)

in the form adapted to the NS5-brane propagating in the $D = 10$ background, let us introduce the NS5 counterpart of the tensor $\hat{\mathcal{H}}_{mn}^*$ (8)

$$\mathcal{H}_{mn}^* = -\frac{2}{\sqrt{-g}} \frac{\delta \mathcal{L}_{kin.NS5}}{\delta H^{*mn}}, \quad (40)$$

where $\mathcal{L}_{kin.NS5}$ denotes the first (DBI-like) term in the action (39), which is just the DBI-like term of the M5-action (1) written in the $D = 10$ adapted worldvolume frame. Using (35), it is easy to find the relation between $\hat{\mathcal{H}}_{mn}^*$ and \mathcal{H}_{mn}^*

$$\mathcal{H}_{mn}^* = \hat{\mathcal{H}}_{mn}^* \sqrt{\frac{(\partial a)^2}{\partial a \partial a}}. \quad (41)$$

Taking into account eqs. (24), (33), (36) and (41) we obtain the following form of the local worldvolume symmetries

$$\delta a = 0, \quad \delta b_{mn} = -2\partial_{[m} a \phi_{n]}(\xi), \quad (42)$$

$$\delta a = \varphi(\xi),$$

$$\delta b_{mn} = \frac{\delta a}{\sqrt{-(\partial a)^2}} \left[\mathcal{H}_{mn}^* - \frac{1}{\mathcal{N}^2} H_{mnp} (g^{ps} + \frac{e^{2\Phi} \mathcal{F}^p \mathcal{F}^s}{1 - e^{2\Phi} \mathcal{F}^2}) \frac{\partial_s a}{\sqrt{-(\partial a)^2}} \right] \quad (43)$$

and the self-duality equation for the NS5-brane gauge field b_2

$$\mathcal{H}_{mn}^* = \frac{1}{\mathcal{N}^2} H_{mnp} (g^{ps} + \frac{e^{2\Phi} \mathcal{F}^p \mathcal{F}^s}{1 - e^{2\Phi} \mathcal{F}^2}) \frac{\partial_s a}{\sqrt{-(\partial a)^2}}, \quad (44)$$

with \mathcal{N} and \mathcal{H}_{mn}^* defined in (37) and (40).

In addition to the worldvolume diffeomorphisms and the symmetries (42) and (43), the action (39) (by construction) has gauge symmetries (5) and (28).

Thus, we have obtained the action describing the worldvolume dynamics of the bosonic 5-brane propagating in the ‘Kaluza–Klein’ part (29) of the IIA $D = 10$ supergravity background. In the next section we extend this action to describe coupling of the NS5-brane to antisymmetric gauge fields of IIA $D = 10$ supergravity.

4 Coupling to the background gauge fields

When the M5-brane couples to the 3-form background field $\hat{C}^{(3)}$ of $D = 11$ supergravity the field strength H_3 gets extended by the worldvolume pullback of $\hat{C}^{(3)}$

$$H^{(3)} \rightarrow \hat{H}^{(3)} = db^{(2)} - \hat{C}^{(3)}. \quad (45)$$

As a result, up to a total derivative, the variation (18) of the action (1) with respect to $b_{mn}(\xi)$ and $a(\xi)$ acquires an additional term in comparison with eq. (21)

$$\int d\xi^6 \delta \mathcal{L} = - \int \left[d(da \wedge \mathcal{F}_2) \wedge (\delta b_2 - \delta a \mathcal{F}_2) + \frac{1}{2} d\hat{C}_3 \wedge \delta b_2 \right], \quad (46)$$

The symmetries (6) and (7) spoiled by the last term of (46) are restored if to the action (1) one adds the Wess-Zumino term [25]

$$S_{WZ} = \int_{\mathcal{M}^6} (\hat{C}^{(6)} + \frac{1}{2} db^{(2)} \wedge \hat{C}^{(3)}), \quad (47)$$

As it was shown in [6], the symmetries (6) and (7) uniquely fix the relative factor between S_{WZ} and the action (1). In (47) $\hat{C}^{(6)}$ is the pullback of a six-form gauge potential whose field strength is $D = 11$ Hodge-dual to the field strength of $\hat{C}^{(3)}$

$$d\hat{C}^{(6)} + \frac{1}{2} \hat{C}^{(3)} \wedge d\hat{C}^{(3)} = *d\hat{C}^{(3)} \quad (48)$$

In addition to the symmetries (6) and (7) with $H^{(3)}$ generalized as in (45), the M5-brane action (1) extended by the Wess-Zumino term (47) is invariant under the following transformations of the antisymmetric gauge fields

$$\delta \hat{C}^{(6)} = d\hat{\varphi}^{(5)} - \frac{1}{2} \delta \hat{C}^{(3)} \wedge \hat{C}^{(3)}, \quad \delta \hat{C}^{(3)} = d\hat{\varphi}^{(2)}, \quad (49)$$

$$\delta b^{(2)} = \hat{\varphi}^{(2)}(\hat{X}(\xi)). \quad (50)$$

To get the form of the coupling of the NS5-brane to the antisymmetric gauge fields of type IIA D=10 supergravity we should dimensionally reduce $\hat{C}^{(3)}$, $\hat{C}^{(6)}$ and the Wess-Zumino term (47) of the M5-brane. The dimensional reduction of $\hat{C}^{(3)}$ produces a ten-dimensional R-R three-form $C^{(3)}$ and an NS-NS two-form $B^{(2)}$

$$\begin{aligned} \hat{C}^{(3)} &= \frac{1}{3!} d\hat{X}^{\hat{l}} \wedge d\hat{X}^{\hat{n}} \wedge d\hat{X}^{\hat{m}} \hat{C}_{\hat{m}\hat{n}\hat{l}}(\hat{X}) = \\ &= \frac{1}{3!} dX^{\underline{l}} \wedge dX^{\underline{n}} \wedge dX^{\underline{m}} C_{\underline{m}\underline{n}\underline{l}}(X) + \frac{1}{2} dX^{\underline{n}} \wedge dX^{\underline{m}} B_{\underline{m}\underline{n}}(X) \wedge (dy - dX^{\underline{l}} A_{\underline{l}}) \equiv \\ &\equiv C^{(3)} + B^{(2)} \wedge \mathcal{F}, \end{aligned} \quad (51)$$

and the dimensional reduction of $\hat{C}^{(6)}$ produces a ten-dimensional five-form $C^{(5)}$ and a six-form $B^{(6)}$ which are dual to $C^{(3)}$ and $B^{(2)}$, respectively,

$$\hat{C}^{(6)} = B^{(6)} + C^{(5)} \wedge \mathcal{F}, \quad (52)$$

the duality relations can be easily derived by the dimensional reduction of eq. (48).

Thus, the field strength of the self-dual gauge field of the NS5-brane coupled to the $D = 10$ background gauge fields is extended as follows

$$H^{(3)} = db^{(2)} - C^{(3)} - B^{(2)} \wedge \mathcal{F}, \quad (53)$$

and the NS5-brane action (39) is enlarged with the following Wess-Zumino term

$$S_{WZ} = \int_{\mathcal{M}_6} \left[B^{(6)} + C^{(5)} \wedge \mathcal{F} + \frac{1}{2} db^{(2)} \wedge C^{(3)} + \frac{1}{2} db^{(2)} \wedge B^{(2)} \wedge \mathcal{F} \right], \quad (54)$$

where $\mathcal{F} = d\xi^m(\partial_m y - A_m)$ is now the worldvolume pullback of the one-form (51).

We have now obtained the action for the NS5-brane propagating in a background of the bosonic sector of type IIA $D = 10$ supergravity

$$\begin{aligned} S = & - \int d^6 \xi e^{-2\Phi} \sqrt{-\det(g_{mn} - e^{2\Phi} \mathcal{F}_m \mathcal{F}_n)} \sqrt{\det \left(\delta_m^n + i \frac{e^\Phi (g_{mp} - e^{2\Phi} \mathcal{F}_m \mathcal{F}_p)}{\mathcal{N} \sqrt{\det(\delta_m^n - e^{2\Phi} \mathcal{F}_m \mathcal{F}_n)}} H^{*np} \right)} \\ & - \frac{1}{4} \int d^6 \xi \sqrt{-g} \frac{1}{\mathcal{N}^2} H^{*mn} H_{mnk} \left(g^{kp} + \frac{e^{2\Phi} \mathcal{F}^k \mathcal{F}^p}{1 - e^{2\Phi} \mathcal{F}^2} \right) \frac{\partial_p a}{\sqrt{-(\partial a)^2}} \\ & + \int_{\mathcal{M}_6} \left(B^{(6)} + C^{(5)} \wedge \mathcal{F} + \frac{1}{2} db^{(2)} \wedge C^{(3)} + \frac{1}{2} db^{(2)} \wedge B^{(2)} \wedge \mathcal{F} \right). \end{aligned} \quad (55)$$

This action is invariant under the worldvolume gauge transformations (5), (42), (43) and (28), with $H^{(3)}$ now having the form (53), and under target-space gauge transformations

$$\begin{aligned} \delta C^{(3)} &= d\varphi^{(2)} + d\varphi^{(1)} \wedge A, & \delta B^{(2)} &= d\varphi^{(1)}, & \delta A &= d\varphi^{(0)}, \\ \delta b^{(2)} &= \varphi^{(2)} - \varphi^{(1)} \wedge dy, & \delta y &= \varphi^{(0)}, \end{aligned} \quad (56)$$

under which $\delta H^{(3)} = 0$, and

$$\begin{aligned} \delta B^{(6)} &= d\varphi^{(5)} + d\varphi^{(4)} \wedge A - \frac{1}{2} d\varphi^{(2)} \wedge C^{(3)} - \frac{1}{2} d\varphi^{(1)} \wedge A \wedge C^{(3)}, \\ \delta C^{(5)} &= d\varphi^{(4)} - \frac{1}{2} d\varphi^{(2)} \wedge B^{(2)} + \frac{1}{2} d\varphi^{(1)} \wedge C^{(3)} - \frac{1}{2} d\varphi^{(1)} \wedge A \wedge B^{(2)}. \end{aligned} \quad (57)$$

Before proceeding with the consideration of the full super-NS5-brane action let us demonstrate how the action of ref. [15, 16] is obtained from eq. (55).

5 NS5-brane action in the second order approximation.

The action of [15, 16] is a second-order approximation in powers of H_{mnk} of the NS5-brane action, with the self-duality condition being regarded as an extra (actually on-shell) constraint. To get the second-order action we should expand (55) in series of $H^{(3)}$ and truncate it down to the second order in $H^{(3)}$ assuming the worldvolume gauge field to be

weak. Since the Wess–Zumino term is already linear and quadratic in H , we shall write down only the “kinetic” part of the action ⁷.

To carry out such a truncation the simplest way is to first truncate the M5–brane action (1) and then perform its dimensional reduction. Up to the second order in $H^{(3)}$ the M5–brane action has the form

$$S = - \int d^6 \xi \sqrt{-\hat{g}} \left[1 - \frac{1}{4} \hat{H}_{mn}^* \hat{H}^{*mn} + \frac{1}{4} \hat{H}^{*mn} H_{mnp} \hat{v}^p + \dots \right] \quad (58)$$

with the self-duality condition (24) reducing to

$$H_{mnl} - \hat{H}_{mnl}^* = 0. \quad (59)$$

Taking into account the expressions

$$\hat{H}_{mnk}^* = \frac{1}{3! \sqrt{-\hat{g}}} \epsilon_{mnkqrs} H^{qrs}$$

and

$$\epsilon_{mnlpqr} \epsilon^{mnlstv} = -(3!)^2 \delta_p^{[s} \delta_q^t \delta_r^{v]},$$

after some algebra one can rewrite (58) in the following form [17]

$$S = - \int d^6 \xi \sqrt{-\hat{g}} \left[1 - \frac{1}{24} H_{mnl} H^{mnl} - \frac{1}{8 \partial a \partial a} \partial_m a (H^{mnl} - \hat{H}^{*mnl}) (H_{nlp} - \hat{H}_{nlp}^*) \partial^p a + \dots \right]. \quad (60)$$

Discarding in (60) the term containing the auxiliary field $a(\xi)$ and the anti-selfdual tensor $H - \hat{H}^*$ (which is zero on the mass shell (59)), and carrying out the direct dimensional reduction of (60) we recover the NS5–brane action of [15, 16]

$$S = - \int d^6 \xi e^{-2\Phi} \sqrt{-\det(g_{mn} - e^{2\Phi} \mathcal{F}_m \mathcal{F}_n)} \left[1 - \frac{1}{24} (e^{2\Phi} H_{mnk} H^{mnk} + 3 \frac{e^{4\Phi}}{1 - e^{2\Phi} \mathcal{F}^2} \mathcal{F}_m H^{mnk} H_{nkp} \mathcal{F}^p) + \dots \right]. \quad (61)$$

⁷We should note that our choice of the dimensionally reduced C_3 and C_5 differs from that in [15, 16], so the Wess–Zumino term in Eq. (55) is related to the one of Refs. [15, 16] by the following field redefinition:

$$\begin{aligned} y &\rightarrow c^{(0)}, & b^{(2)} &\rightarrow a^{(2)}, \\ B^{(2)} &\rightarrow B^{(2)}, & C^{(3)} - B^{(2)} \wedge A &\rightarrow C^{(3)}, \\ C^{(5)} &\rightarrow C^{(5)} - \frac{1}{2} C^{(3)} \wedge B^{(2)}, & B^{(6)} - C^{(5)} \wedge A &\rightarrow -\tilde{B}^{(6)}. \end{aligned}$$

The WZ term of [15, 16] also contains the curl of an auxiliary worldvolume 5-form field which ensures the exact gauge invariance of the WZ term.

Alternatively, the action (61) can be obtained directly by truncating the NS5-brane action (55), and discarding terms containing the auxiliary field and the linearized NS5-brane self-duality condition (44)

$$H_{mnl}^* - H_{mnp} \left(\delta_l^p + \frac{e^{2\Phi} \mathcal{F}^p \mathcal{F}_l}{1 - e^{2\Phi} \mathcal{F}^2} \right) = 0.$$

6 The κ -symmetric super-NS5-brane action

To generalize the results of previous sections to describe the propagation of an NS5-brane in a curved IIA $D = 10$ target superspace parametrized by ten bosonic coordinates $X^{\underline{m}}$ and 32-component Majorana-spinor fermionic coordinates Θ^α forming a IIA, $D = 10$ superspace coordinate system

$$Z^{\underline{M}} = (X^{\underline{m}}, \Theta^\alpha), \quad (62)$$

we again start with an M5-brane propagating in a generic D=11 supergravity background parametrized by eleven bosonic coordinates $\hat{X}^{\hat{m}}$ and 32-component Majorana-spinor fermionic coordinates Θ^α forming a $D = 11$ superspace coordinate system

$$\hat{Z}^{\hat{M}} = (\hat{X}^{\hat{m}}, \Theta^\alpha) = (Z^{\underline{M}}, y), \quad (63)$$

where we have separated the eleventh coordinate $y = X^{10}$ keeping in mind the dimensional reduction of $D = 11$ superspace down to type IIA $D = 10$ superspace.

$D = 11$ superspace geometry is described by a supervielbein

$$\hat{E}^{\hat{A}} = d\hat{Z}^{\hat{M}} \hat{E}_{\hat{M}}^{\hat{A}}(\hat{Z}) = (\hat{E}^{\hat{a}}, \hat{E}^\alpha), \quad (64)$$

where $\hat{A} = (\hat{a}, \alpha)$ are locally flat tangent superspace indices, by a superconnection

$$\hat{w}_{\hat{A}}^{\hat{B}} = d\hat{Z}^{\hat{M}} \hat{w}_{\hat{M}\hat{A}}^{\hat{B}}(\hat{Z}), \quad (65)$$

and by a three-superform generalization of the bosonic gauge field (51)

$$\hat{C}^{(3)} = \frac{1}{3!} d\hat{Z}^{\hat{N}} \wedge d\hat{Z}^{\hat{M}} \wedge d\hat{Z}^{\hat{L}} C_{\hat{L}\hat{M}\hat{N}}(\hat{Z}). \quad (66)$$

The supervielbein, the superconnection and the gauge superfield are subject to supergravity constraints which put the superfield formulation of eleven-dimensional supergravity on the mass shell. An explicit form of the $D = 11$ supergravity constraints relevant to the description of M5-brane dynamics the reader may find in [29, 7, 23, 24].

The super-M5-brane action has the similar form as the bosonic action (1) enlarged with the WZ term (47), where the worldvolume induced metric is now

$$\hat{g}_{mn} = \partial_m \hat{Z}^{\hat{M}} \partial_n \hat{Z}^{\hat{N}} \hat{E}_{\hat{N}}^{\hat{a}}(\hat{Z}) \hat{E}_{\hat{M}\hat{a}}(\hat{Z}), \quad (67)$$

and $\hat{C}^{(3)}$ and $\hat{C}^{(6)}$ are worldvolume pullbacks of the three-superform (66) and its six-superform dual [29, 30].

In addition to all symmetries discussed above and target-space superdiffeomorphisms the super-M5-brane action is invariant under the following fermionic κ -symmetry transformations [7, 8]

$$i_\kappa \hat{E}^{\hat{a}} \equiv \delta_\kappa \hat{Z}^{\hat{M}} \hat{E}_{\hat{M}}^{\hat{a}}(\hat{Z}) = 0, \quad i_\kappa \hat{E}^{\hat{a}} \equiv \delta_\kappa \hat{Z}^{\hat{M}} \hat{E}_{\hat{M}}^{\hat{a}}(\hat{Z}) = (I - \bar{\Gamma})^{\hat{a}\hat{b}} \kappa_{\hat{b}}, \quad (68)$$

$$\delta_\kappa b_2 = i_\kappa \hat{C}_3 \equiv \frac{1}{2} d\hat{Z}^{\hat{M}_3} \wedge d\hat{Z}^{\hat{M}_2} \delta_\kappa \hat{Z}^{\hat{M}_1} \hat{C}_{\hat{M}_1 \hat{M}_2 \hat{M}_3}(\hat{Z}), \quad \delta_\kappa a = 0.$$

where the spinor matrix $\bar{\Gamma}$ has the following expansion in products of $D = 11$ Dirac matrices

$$\bar{\Gamma} = \frac{\sqrt{-\hat{g}}}{\sqrt{-\det(\hat{g} + i\hat{H}^*)}} \left(\hat{\Gamma}^{(6)} + \frac{i}{2} \hat{H}_{mn}^* \hat{v}_l (\hat{\Gamma}^{mnl}) + \hat{t}_m \hat{v}_n (\hat{\Gamma}^{mn}) \right) \quad (69)$$

$$\hat{\Gamma}^{(6)} = \frac{1}{6!} \varepsilon^{m_1 \dots m_6} \hat{\Gamma}_{m_1} \dots \hat{\Gamma}_{m_6}, \quad \hat{\Gamma}_m \equiv \partial_m \hat{Z}^{\hat{M}} \hat{E}_{\hat{M}}^{\hat{a}}(\hat{Z}) \hat{\Gamma}_{\hat{a}},$$

$$\hat{t}^m = \frac{1}{8} \varepsilon^{mnkplq} \hat{H}_{nk}^* \hat{H}_{pl}^* \hat{v}_q \equiv \frac{1}{8} \varepsilon^{mnkplq} \hat{\mathcal{H}}_{nk}^* \hat{\mathcal{H}}_{pl}^* \hat{v}_q.$$

As is characteristic of all superbranes, for the M5-brane action to be κ -symmetric the superbackground must satisfy the supergravity constraints [29, 7, 24]. When they are taken into account, from (68) we get

$$\delta_\kappa \hat{H}_3 = -i_\kappa \hat{F}_4 = \hat{E}^{\hat{a}} \wedge \hat{E}^{\hat{b}} \wedge \hat{E}^{\hat{\alpha}} \left(\Gamma_{\hat{a}\hat{b}} (I - \bar{\Gamma}) \right)_{\underline{\alpha}\underline{\beta}} \kappa_{\underline{\beta}}, \quad (70)$$

$$\delta_\kappa \hat{g}_{mn} = -4i \hat{E}_{(m}^{\hat{\alpha}} \left(\Gamma_{n)} (I - \bar{\Gamma}) \right)_{\underline{\alpha}\underline{\beta}} \kappa_{\underline{\beta}}.$$

We now turn to the consideration of the super-NS5-brane action. It can be obtained from the super-M5-brane action by the direct dimensional reduction of the $D = 11$ supergravity superfields. A consistent ansatz for the dimensionally reduced supervielbein (64) was proposed in [32]. This is the following superfield generalization of eq. (27)

$$\hat{E}^{\hat{a}} = e^{-\frac{1}{3}\Phi(Z)} E^{\hat{a}}, \quad \hat{E}^{10} = e^{\frac{2}{3}\Phi(Z)} (dy - dZ^{\hat{M}} A_{\hat{M}}(Z)) \equiv e^{\frac{2}{3}\Phi(Z)} \mathcal{F}, \quad (71)$$

$$\hat{E}^{\hat{\alpha}} = e^{-\frac{1}{6}\Phi(Z)} E^{\hat{\alpha}}(Z) + \mathcal{F} \chi^{\hat{\alpha}}(Z), \quad (72)$$

where $E^{\hat{A}}(Z) = dZ^{\hat{M}} E_{\hat{M}}^{\hat{A}} = (E^{\hat{a}}, E^{\hat{\alpha}})$ are supervielbeins of type IIA $D = 10$ supergravity, $\Phi(Z)$ is the dilaton superfield, $A_{\hat{M}}(Z)$ are components of the one-form gauge superfield $A = dZ^{\hat{M}} A_{\hat{M}}(Z)$, and $\chi^{\hat{\alpha}}(Z)$ is a Grassmann-odd Majorana spinor superfield, which is actually the Grassmann derivative of the dilaton superfield $\Phi(Z)$.

The superfields which describe IIA $D = 10$ supergravity are subject to the constraints which are obtained from the $D = 11$ supergravity constraints using the ansatz (71), (72)

and solving for Bianchi identities. Different forms of these constraints have been considered in [33], [32, 34, 35].

We do not write the super-NS5-brane action explicitly since it has exactly the same form as Eq. (55) where now the worldvolume induced metric is

$$g_{mn} = \partial_m Z^{\underline{M}} \partial_n Z^{\underline{N}} E_{\underline{N}}^{\underline{a}}(Z) E_{\underline{M}\underline{a}}(Z), \quad (73)$$

and all the bosonic background fields are replaced with corresponding superfields, in particular, $B^{(6)}$, $C^{(5)}$, $C^{(3)}$ and $B^{(2)}$ are the worldvolume pullbacks of the type IIA $D = 10$ superforms

$$C^{(n)}(Z) = \frac{1}{n!} E^{\underline{A}_n} \wedge \dots \wedge E^{\underline{A}_1} C_{\underline{A}_1 \dots \underline{A}_n}(Z). \quad (74)$$

Note that the spinor superfield χ^α does not appear in the action (55).

The super-NS5-brane action is invariant under κ -symmetry transformations obtained from eqs. (68) by substituting into the latter the ansatz (71) and (72)

$$i_\kappa E^{\hat{\underline{a}}} \equiv \delta_\kappa Z^{\underline{M}} E_{\underline{M}}^{\underline{\alpha}}(Z) = (I - \bar{\Gamma})^{\underline{\alpha}\underline{\beta}} \kappa_{\underline{\beta}}, \quad (75)$$

$$i_\kappa E^{\underline{a}} = 0, \quad i_\kappa \mathcal{F} = 0 \quad \Rightarrow \quad \delta_\kappa y = i_\kappa E^{\underline{\alpha}} A_{\underline{\alpha}}(Z), \quad (76)$$

$$\delta_\kappa b_2 = i_\kappa C_3 + \mathcal{F} \wedge i_\kappa B_2, \quad \delta_\kappa a = 0.$$

where $A_{\underline{\alpha}}(Z)$ is a fermionic component of the Kaluza-Klein connection form

$$A \equiv dZ^{\underline{M}} A_{\underline{M}} = E^{\underline{\alpha}} A_{\underline{\alpha}} + E^{\underline{a}} A_{\underline{a}}.$$

7 Conclusion and Discussion

To summarize, we have obtained the covariant κ -symmetric action for the super-NS5-brane in a IIA $D = 10$ supergravity background by the direct dimensional reduction of the M-theory super-five-brane action. In addition to worldvolume diffeomorphisms, gauge symmetry, κ -symmetry and background supergravity symmetries the super-NS5-brane action possesses special local symmetries ensuring the covariance of actions with self-dual gauge fields and serving for deriving the self-duality condition directly from the action as a consequence of the equation of motion of the gauge field.

An interesting problem for future study is to construct the Lagrangian description of the consistent coupling of a type IIA supergravity action to an NS5-brane source. The latter requires the construction of a duality-symmetric version of type IIA supergravity by the dimensional reduction of the duality-symmetric $D = 11$ supergravity [30]. The truncation of such a IIA supergravity action shall produce the duality-symmetric version of the $N = 1$, $D = 10$ supergravity, which should naturally couple to a heterotic five-brane [36]. Note that recent investigations of interacting brane actions [37] may provide one with a possibility of making this coupling supersymmetric.

Another problem for further studying is to perform the T-duality transformation of the complete NS5-brane action and to arrive at a non-linear and supersymmetric action for a type IIB D=10 Kaluza-Klein (KK) monopole. A quadratic approximation for the bosonic part of this action has been constructed in [16]. One of possible ways of deriving appropriate T-duality transformation rules for the antisymmetric gauge fields is to T-dualize the duality-symmetric version of type IIA supergravity to the duality-symmetric version of type IIB supergravity [38].

As it was noted in [31] and proved in the second order approximation in [16], the type IIB D=10 KK monopole is expected to be a self-dual object under the S-duality symmetry of type IIB supergravity. The construction of the complete action for the type IIB KK monopole should allow one to explicitly verify this statement.

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